

PHENOMENOLOGY OF THE ELECTRON STRUCTURE FUNCTION*

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Abstract

Advantages of introducing the electron structure function (ESF) in electron induced processes are demonstrated. Contrary to the photon structure function it is directly measured in such processes. At present energies a simultaneous analysis of both the electron and the photon structure functions gives an important test of the experimentally applied methods. Estimates of the ESF at currently measured momenta are given. At very high momenta contributions from W and Z bosons together with γ - Z interference can be observed. Predictions for next generation of experiments are given.

In a series of papers [1, 2] we have presented the construction of the electron structure function — a useful notion in the QCD analysis of electron induced hadron production. The Q^2 evolution equations have been constructed and asymptotic solutions found for the quark and gluon content of the electron in the leading logarithmic approximation. We included contributions from all intermediate bosons, in particular we found that the γ - Z interference is important at very high energies. This spoils the usual probabilistic interpretation of separate $\gamma/Z/W$ structure functions. We also found that in certain experimental situations the commonly used convolution of the photon structure function and the photon flux, used to describe the electron induced processes, is incorrect.

In this letter we study phenomenology of the electron structure function comparing it with the well known approach which makes use of the photon structure function. Theoretical framework which allows to calculate the photon structure is known since long [3]. It appears as perturbative QCD contribution, in addition to the modelled Vector Meson

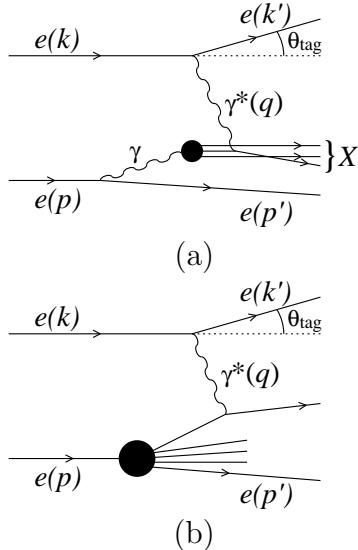


Figure 1: Deep inelastic scattering on a photon (a) and electron (b) target

Dominance term. To measure this photonic structure, experiments [4] use the electron (or positron) beam as a source of photons. Despite precise measurement the data are not easy to extract. The problem is displayed in Fig. 1a. The tagged (upper) electron emits a probing photon whereas the untagged (lower) one goes nearly along the beam, emitting the target photon. (The situation where at very high energies the probing boson can be also a Z boson is considered below.) The large scale Q^2 is determined by the tagged electron:

$$Q^2 = -(k - k')^2 = 2EE_{\text{tag}}(1 - \cos \theta_{\text{tag}}), \quad (1)$$

where E is the initial electron energy and E_{tag} and θ_{tag} are the energy and polar angle of the measured electron. The antitag condition (if present) requires the virtuality of the target photon P^2 to be less

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than certain P_{\max}^2 :

$$-(p - p')^2 \equiv P^2 \leq P_{\max}^2. \quad (2)$$

This photon is clearly not a beam particle and has the energy diffused according to the equivalent photon spectrum:

$$f_\gamma^e(y_\gamma, P^2) = \frac{\alpha}{2\pi P^2} \left[\frac{1 + (1 - y_\gamma)^2}{y_\gamma} - \frac{2y_\gamma m_e^2}{P^2} \right] \quad (3)$$

where y_γ is the photon momentum fraction, α is the QED structure constant, and m_e is the electron mass. The measured cross-section for the production of a hadronic system X , expressed in terms of the photon structure functions F_2^γ and F_L^γ reads:

$$\frac{d^3\sigma_{ee \rightarrow eX}}{dz dQ^2 dx} = \frac{2\pi\alpha^2}{x^2 Q^4} \int_{P_{\min}^2}^{P_{\max}^2} [(1 + (1 - y)^2) F_2^\gamma(x, Q^2, P^2) - y^2 F_L^\gamma(x, Q^2, P^2)] f_\gamma^e(y_\gamma, P^2) dP^2 \quad (4)$$

where

$$\begin{aligned} y &= 1 - (E_{\text{tag}}/E) \cos^2(\theta_{\text{tag}}/2), \\ P_{\min}^2 &= m_e^2 y_\gamma^2 / (1 - y_\gamma) \end{aligned} \quad (5)$$

and x (z) are fractions of the parton momentum with respect to the photon (electron). They are related to the photon momentum fraction with respect to the electron by

$$z = xy_\gamma \quad (6)$$

The integral over the photon virtuality is usually performed by assuming $P^2 = 0$ in the photon structure functions (Weizsäcker-Williams approximation [7]) which leads to:

$$\begin{aligned} \frac{d^3\sigma_{ee \rightarrow eX}}{dz dQ^2 dx} &= \frac{2\pi\alpha^2}{x^2 Q^4} \\ &\times [(1 + (1 - y)^2) F_2^\gamma(x, Q^2, 0) - y^2 F_L^\gamma(x, Q^2, 0)] \\ &\times f_\gamma^{\text{WW}}(z/x, P_{\max}^2) \end{aligned} \quad (7)$$

where

$$\begin{aligned} f_\gamma^{\text{WW}}(y_\gamma, P_{\max}^2) &= \frac{\alpha}{2\pi} \left[\frac{1 + (1 - y_\gamma)^2}{y_\gamma} \ln \frac{P_{\max}^2(1 - y_\gamma)}{m_e^2 y_\gamma^2} \right. \\ &\left. - 2 \frac{1 - y_\gamma}{y_\gamma} \right] \end{aligned} \quad (8)$$

This is how the real photon structure functions F_2 , F_L appear in lepton-lepton scattering.

Three remarks are important for further considerations. First, the splitting of the process into a

distribution of photons inside electron f_γ^{WW} and that of partons inside the photon F_2^γ is an approximation. The optimal form of the equivalent photon formula is still being discussed [8]. Even if most of experimental groups choose the same formula, one should keep in mind that the photon structure function depends on this convention.

Second, the target photon is always off-shell and its virtuality is experimentally not measured. Although the equivalent photon distribution is peaked at minimum (nearly zero) virtuality, treating the photon as real is another approximation. One should keep in mind that the measured photon structure function depends on x , Q^2 and P^2 , and the, usually neglected, P^2 dependence can be quite strong [12]. A hint that we are not measuring the real photon structure function comes also from the analysis of the QED structure function of the photon where analytical solutions (and thus the P^2 dependence) are known [5]. The data (coming from $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ scattering [6]) agree with theory only if non-zero virtuality is taken into account. In the case of QCD the situation is more difficult — experiments yield the photon structure function at virtuality P^2 which is not measured and theory does not predict this P^2 dependence.

Third, in order to fix x , one is forced to measure — in addition to the tagged electron — the hadronic momenta. In fact,

$$x = \frac{Q^2}{Q^2 + P^2 + W^2} \approx \frac{Q^2}{Q^2 + W^2}, \quad (9)$$

where W is the invariant mass of the produced hadronic system X . Its determination is more difficult than of other (tagged electron) variables since substantial part of hadrons is lost in the beam pipe. The uncertainty in the determination of the x variable is the source of large uncertainties in the analysis (unfolding procedure). The data are indirectly biased by theoretical assumptions.

Many of the above problems can be avoided when we introduce the structure function of the electron (Fig. 1b). To see how it works let us first write the cross-section in terms of the electron structure functions F_2^e and F_L^e :

$$\begin{aligned} \frac{d^2\sigma_{ee \rightarrow eX}}{dz dQ^2} &= \frac{2\pi\alpha^2}{z Q^4} [(1 + (1 - y)^2) F_2^e(z, Q^2, P_{\max}^2) \\ &- y^2 F_L^e(z, Q^2, P_{\max}^2)]. \end{aligned} \quad (10)$$

The structure function $F_2^e(z, Q^2, P_{\max}^2)$ which dominates the cross-section at small y , has simple

partonic interpretation:

$$F_2^e(z, Q^2, P_{\max}^2) = z \sum_i e_{q_i}^2 q_i(z, Q^2, P_{\max}^2), \quad (11)$$

where e_{q_i} and q_i are the i -th quark fractional charge and density. This is a standard deep inelastic scattering process where the cross-section is related to the structure function via simple kinematical factors. More precisely the above defined electron structure function corresponds exactly to the proton structure function if no antitag condition is imposed and P_{\max}^2 goes up to its kinematical limit of the order of Q^2 . (This is the ‘inclusive case’ in our terminology [1], the electron structure function depends then on z and Q^2 only.)

The argument z — the parton momentum fraction with respect to the electron — is measured, as in the standard deep inelastic scattering, by means of the tagged electron variables only:

$$z = \frac{Q^2}{2pq} = \frac{\sin^2(\theta_{\text{tag}}/2)}{E/E_{\text{tag}} - \cos^2(\theta_{\text{tag}}/2)}. \quad (12)$$

There is no need *a priori* to reconstruct the hadronic mass W . In present experimental analyses one introduces in both the photon and electron structure analyses lower limit on W because the reconstruction of the hadronic mass is unreliable below this limit. But even so, there is an important difference between the case when W (in small W region) is used only as a selection cut to reduce background (electron structure) and the case when in addition it is used to determine the x variable (photon structure). In particular a lower cut on W is much less sensitive to the unfolding procedure than the value of W itself. The quantitative analysis of W influence is discussed below.

All these features cause that the same experiment can produce more precise and analysis independent data when looking at the electron structure. What is most important — the electron structure function contains the same information about QCD as the photon one and it is known theoretically with at least the same accuracy. Moreover, it allows to avoid problems which arise in the photon structure function at very high energies.

At present energies, where the W and Z bosons contributions are negligible, one can reanalyse the existing data in terms of the electron structure function. This can be treated also as a consistency check of both photon and electron structure. Phenomenologically, having a parametrisation of the photon structure function which describes well the

existing data, we can predict the electron structure function for $Q^2 \gg P_{\max}^2$ by taking the convolution of this parametrisation with the equivalent photon spectrum:

$$F_2^e(z, Q^2, P_{\max}^2) = \int_z^1 dy_\gamma \int_{P_{\min}^2}^{P_{\max}^2} dP^2 f_\gamma^e(y_\gamma, P^2) \times F_2^\gamma \left(\frac{z}{y_\gamma}, Q^2, P^2 \right) \quad (13)$$

Such convolution is correct when the experiments use antitag condition (‘exclusive case’ in our terminology [1]). The curves with momenta corresponding to LEP2 and TESLA/NLC/JLC experiments, resulting from some popular parametrisations [10, 11] of real photon structure — *i.e.* $P^2 = 0$ in F_2^γ of Eq. (13) — are shown in Fig. 2. To test how significant is the (neglected above) P^2 dependence we also plot the resulting curve of SaS-1D parametrisation with the P^2 dependence built in [11]. In the latter case the integration over P^2 in Eq.(13) is performed numerically. Our previous arguments are confirmed: already at LEP2 the photon virtuality is non-negligible, it can produce effects of the same order as differences between various parametrisations. The electron variables z, Q^2 and P_{\max}^2 are well known, the photon data do not contain one crucial information — the photon virtuality, at which the structure is measured.

As noticed in Ref. [13], the shape of the electron structure function is strongly influenced by the QED part (“photon flux”). All F_2^e functions resulting from various photon structure parametrisations are falling functions of z and look “similar”. In addition the contributions at given electron momentum fraction $z = xy_\gamma$ come both from large x and small y_γ as well as large y_γ and small x . Both these features can be regarded as drawbacks of the electron structure function. The above arguments are only partly true. First, one should keep in mind that the variables x, y_γ and z are not independent and their interplay under the integral Eq. (13) is specific. To see the problem in more detail let us take the Eq. (13) within the Weizsäcker-Williams approximation (P_{\max}^2, P^2, Q^2 suppressed)

$$F_2^e(z) = \int_z^1 dy_\gamma f_\gamma^{\text{WW}}(y_\gamma) F_2^\gamma \left(\frac{z}{y_\gamma} \right) \quad (14)$$

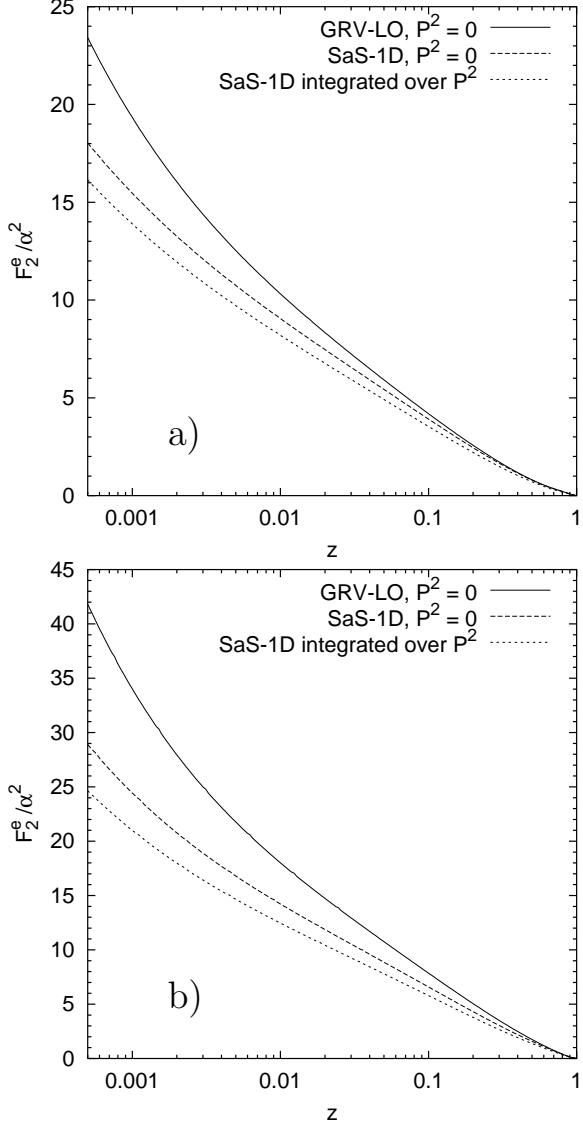


Figure 2: Predicted value of the electron structure function $F_2^e(z)/\alpha^2$ at: a) $Q^2 = 17.8 \text{ GeV}^2$, $P_{\max}^2 = 4.5 \text{ GeV}^2$ and b) $Q^2 = 120 \text{ GeV}^2$, $P_{\max}^2 = 30 \text{ GeV}^2$ from different parametrisations [10, 11] of the photon structure function: SaS-1D (broken line), GRV-LO (solid line).

Noting that essentially $f_\gamma^{\text{WW}}(y_\gamma) \propto 1/y_\gamma$ we get from Eq. (14):

$$F_2^e(z) \propto \int_z^1 \frac{dy_\gamma}{y_\gamma} F_2^\gamma \left(\frac{z}{y_\gamma} \right) = \int_z^1 \frac{dx}{x} F_2^\gamma(x). \quad (15)$$

One sees that in this approximation the z dependence comes via the lower limit of integration.

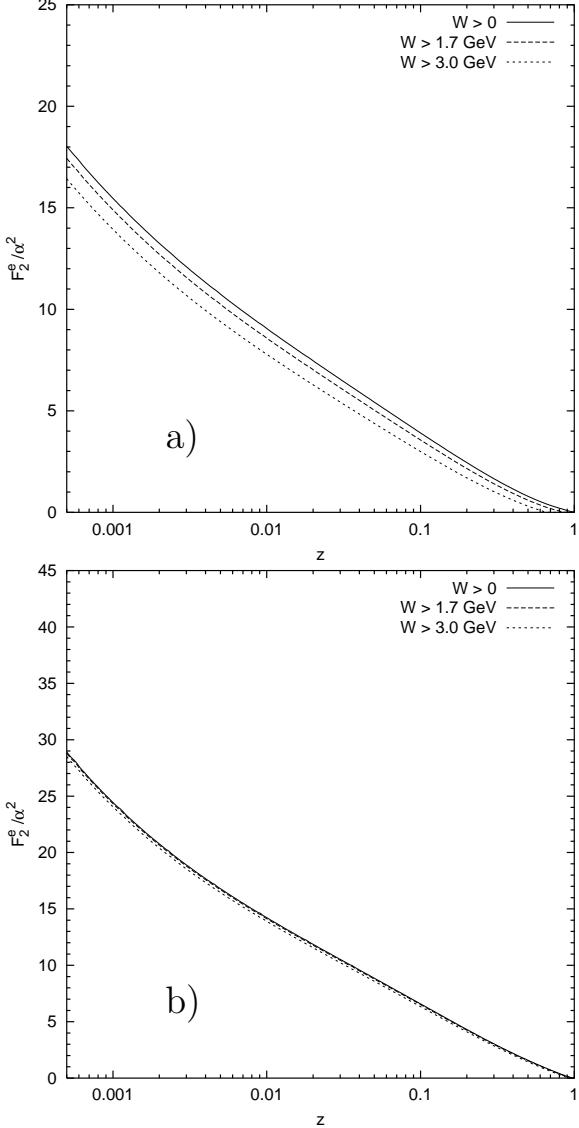


Figure 3: Predicted value of the electron structure function $F_2^e(z)/\alpha^2$ from SaS-1D parametrisation (solid line). The effect on measured values when a cut on W is imposed: $W \geq 1.7 \text{ GeV}$ — dashed line, $W \geq 3.0 \text{ GeV}$ — dotted line. a) $Q^2 = 17.8 \text{ GeV}^2$, $P_{\max}^2 = 4.5 \text{ GeV}^2$; b) $Q^2 = 120 \text{ GeV}^2$, $P_{\max}^2 = 30 \text{ GeV}^2$.

Moreover the importance of the small x region under the integral (14) is enhanced by the $1/x$ factor. Second, due to the same kinematics the data points are generally shifted towards lower z (as compared to x). Therefore the experimental results are more accurate at small z , a feature mostly welcome in the region where new effects are to be expected. In

addition, we remind that the photon data points at low x are strongest influenced by the unfolding procedure.

As already mentioned, the present way of data analysis introduces a lower cut on the hadronic mass W . In the photon case it causes that we do not measure the photon structure function above certain x (see Eq. (9)). In the measurement of the electron structure function W is not needed at all (as it is not used in the analysis of the proton structure function). A cut on W imposed in present experiments lowers the cross-section in the whole z range, see Fig. 3. We checked that *e.g.* for $Q^2 = 17.8 \text{ GeV}^2$ the effect of the condition $W \geq 1.7 \text{ GeV}$ (with SaS-1D parametrisation in Eq. (13)) is less than 5% for $z \leq 0.01$, and less than 9% for $z \leq 0.1$. This suppression gets smaller with growing Q^2 (*e.g.* at $Q^2 = 120 \text{ GeV}^2$ it is below 1% for $z \leq 0.1$).

The concept of the electron structure function introduces new interesting effects at momenta much higher than presently available. One has then to take into account not only the photon flux contributing to the electron structure but also those resulting from Z and W bosons. As shown in Ref. [1] γ - Z interference comes into play and is comparable to the Z contribution itself. This means that

a notion of separate gauge boson structure functions (γ , Z or W) loses sense and only the electron structure function preserves probabilistic interpretation. The question is, can we observe these effects in the next generation of experiments? In Fig. 4 we give quantitative estimate in the case of single tag e^+e^- scattering at CLIC [9] momenta, choosing $Q^2 = 10000 \text{ GeV}^2$ and $P_{\max}^2 = 1000 \text{ GeV}^2$. In this case the picture of Fig. 1 gets modified. The upper (tagged) electron emits now both the photon and the Z boson. In the calculation of the electron structure one has to take into account contributions from the photon, Z boson, their interference (with the antitag condition fixed by P_{\max}^2) and the W boson (no antitag condition). The presented curves are asymptotic solutions of our electron evolution equations [1]. One sees that the effect of Z and γ - Z terms is of the order of 5 – 15%. We checked that it can be enhanced to 20 – 25% in a double tag experiment.

Let us add a few final remarks. First concerns the study of the virtual photon structure [12] (double tag experiment). The analysis can be reformulated in terms of the P_{\max}^2 dependence of the electron structure function. Studying a real, convention independent object is first advantage. Another one is the fact that at very high virtualities the Z admixture and the γ - Z interference are properly taken into account.

Second is a comment on the QED structure function of the photon. It is obtained from the process $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ by dividing out the (approximate) equivalent photon distribution and assuming some effective photon virtuality. The use of the QED electron structure function avoids the above approximations. The exactly known (in given order of α) electron structure function can be compared directly with the electron data.

Finally, the photon structure has been also measured [14] in dijet production at HERA. Again the extraction of the x variable is difficult. In addition to jets, one has to measure essentially the whole hadronic system in order to obtain the photon energy. The data, when presented in terms of the electron structure, require only measurement of the two jets. Practically the new approach means plotting the dijet cross-section as a function of z . A more ambitious program would be to extract the parton densities in the electron, as it has been done in the case of the photon [15] or to construct a parametrisation of parton densities inside the electron by a direct fit to the HERA data.

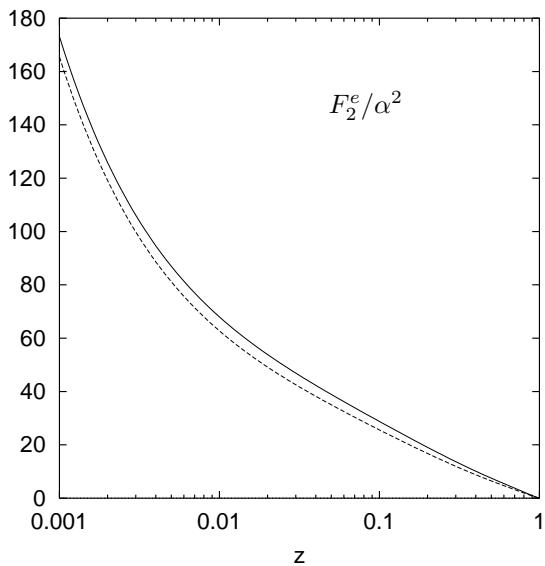


Figure 4: The structure function $F_2^e(z, Q^2, P_{\max}^2)/\alpha^2$ at $Q^2 = 50000 \text{ GeV}^2$ and $P_{\max}^2 = 1000 \text{ GeV}^2$ (solid line). The contribution from the photon only is also shown (broken line).

To summarise, we propose to look at the electron as surrounded by a QCD cloud of quarks and gluons (in order α^2), very much like it is surrounded by a QED cloud of equivalent photons (in order α). We argue that the use of the electron structure function in electron induced processes has some advantages over the photon one. Experimentally it leads to more precise, convention independent data. Theoretically it allows for more careful treatment of all variables. It also takes into account all electroweak gauge boson contributions, including their interference, which will be important in the next generation of e^+e^- colliders. At present energies it should certainly be used as a cross-check of the photon structure analysis.

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